P014 CONNECTION BETWEEN POISSON RATIO AND MICROSTRUCTURE IN MICROINHOMOGENEOUS MEDIA

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ABSTRACT.

A new approach to describe physical-deforming processes in microinhomogeneous media is presented. This approach is based on examination of individual interactions between particles and used a new method of field forces averaging for chaotic orientation of neighbors. It is possible to create equations of motion in acoustic wave approximation. This approach may be useful for connection between micro (structure of forces which are acting between grains) and macro (Poisson ratio, velocities of P and S waves) parameters of media for sandy media (it is not necessary construct by spheres of identical diameter), clays of cellular structure with dipole law interaction between grains, the consolidated granular environments without porosity. It is established that dependence of the velocities relation P and S waves on the ratio between normal and tangent forces, which are acting at a micro level. Besides of it, made an explanation of the growing of the velocity of P waves by growing of density in materials with nonmetallic connection between grains and the fall of it with in the case of metals. There is satisfactory confirmation between theory and well-known experimental results.

Key words: microinhomogeneous media, sands-clay sediments, elastic waves, microstructure, procedure of averaging.

INTRODUCTION.

In EAGE Conference (Amsterdam, 2001) was presented a new method of averaging of field forces, acting between grains in contrast microinhomogeneous medium [1]. Equations of motion, which was created by this method, are satisfactory described the propagation of elastic waves in sands and given the dependence of elastic module as a function of microstructure parameters. This method was given for two-dimensional space only and for media with Hertz law interaction of identical spheres. Now this procedure of averaging is generalized for three-dimensional space. Some special cases of this method application are considered.

Sands.

It is imaginable the sandy environment as a set of spheres with the known law of distribution of diameters. Improvement of the approach established in [1] consists on averaging of forces not in a plane, but in the volume. This medium is submitted as a set of mesostructures (Fig 1). Mesostructure consists of the central grain and platforms of contacts of its neighbors. Platforms have chaotic orientation. One grain may differ from another in radius and orientation of normal to platforms of contacts. The law of distribution of radiuses F (R) is supposed as a known thing. Orientation of mesostructures in the medium has the uniform distribution. The averaging procedure is realized not by a plane angle, but by solid angle (θ and φ - are polar and azimuthal angles). There is a possibility to write Hertz force acting on a random sphere due to its neighbors (N - an average of contacts):

$F_n = -A\delta_n^{3/2}$, (1)

where $A = \dfrac{2E}{3(1-\nu^2)}\left(\dfrac{R_1 R_2}{R_1 + R_2}\right)^{1/2}$,

$\delta_n = \delta_0 - \Delta U\cos\varphi\sin\theta - \Delta V\sin\varphi\sin\theta - \Delta W\cos\theta$, (2)

δ_0 – initial displacement from equilibrium state due to initial (static) pressure, E – Young modulus, ν - Poisson ratio, R_1 – radius of the random grain, R_2 – radius of its neighbor, ΔU, ΔV, ΔW are displacements of grains centers from equilibrium state with respect to axes X, Y and Z. Radiuses R_1 and R_2 are distributed by the known law. Replacing difference operators by differential (and using a condition, that $\delta_0 >> \Delta U, \Delta V, \Delta W$), it is possible to receive the three-dimensional equations of the theory of elasticity with the elasticity modules, which are equal:

$$\lambda+2\mu= \frac{3E(1-f)N\delta_0^{1/2}}{40\pi\rho(1-\nu^2)}\frac{1}{\overline{R^2}}\int_0^\infty\int_0^\infty\frac{\sqrt{R_1R_2}(R_1+R_2)^{3/2}}{R_1^3}F(R_1)F(R_2)dR_1dR_2 \quad (3)$$

$$\mu= \frac{E(1-f)N\delta_0^{1/2}}{40\pi\rho(1-\nu^2)}\frac{1}{\overline{R^2}}\int_0^\infty\int_0^\infty\frac{\sqrt{R_1R_2}(R_1+R_2)^{3/2}}{R_1^3}F(R_1)F(R_2)dR_1dR_2, \quad (4)$$

where f – porosity, ρ - density of medium, $\overline{R}$ - average radius of grain. Relation of
$(V_s/V_p)^2 = 1/3$, (5)
and does not depend on radius distribution law. Formula (3) causes a well-known result about 1/6 degree of P wave velocity due to initial static pressure.
Special cases:
Radiuses are uniformly distributed from R to 2R:

$$\lambda+2\mu= \frac{3E(1-f)N\delta_0^{1/2}}{40\pi\rho(1-\nu^2)}\frac{1}{R^2}\int_R^{2R}\int_R^{2R}\frac{\sqrt{R_1R_2}(R_1+R_2)^{3/2}}{R_1^3}dR_1dR_2 \quad (6)$$

$$\mu= \frac{E(1-f)N\delta_0^{1/2}}{40\pi\rho(1-\nu^2)}\frac{1}{R^2}\int_R^{2R}\int_R^{2R}\frac{\sqrt{R_1R_2}(R_1+R_2)^{3/2}}{R_1^3}dR_1dR_2 \quad (7)$$

Identical radiuses:

$$\lambda+2\mu= \frac{3\sqrt{2}E(1-f)N\sqrt{\delta_0 R}}{20\pi\rho(1-\nu^2)},\ \mu=(\lambda+2\mu)/3 \quad (8)$$

Thus, the relation of P and S waves velocities do not depend on grains radiuses at Hertz interaction. It depends on the relation of the normal and tangent forces acting at a micro level. If introduced like [1] the force of dry friction is appear the attenuation, which is proportional to the first degree of a frequency.

Clays.

In clays with cellular structure, where grains are plane elliptic plates, the force picture will be another. Let's suppose, that there is no mechanical contact between particles, and there are the dipole interactions between them only, while there is some initial pressure, which causes initial distances between grains equal to δ. On the Fig 2 is shown a mesostructure of clay medium. Forces of interactions are contains not only normal components only, but also tangents once too. It is a reason why the relation of velocities is another, compared to sands. In spite of it, procedure of averaging is precisely same, as for sand. The calculation of forces, acting on a random particle is doing before of the averaging by orientation of mesostructures. The relation of P and S waves velocities is differ than in sands:

$V_p^2=8ND^2r/(5\pi m\delta^4_0)$ (9)

$\gamma\ (V_s/V_p)=1/\sqrt{11}$ (10)

where D is dipole moment, r – average maximum size of grains, m – average mass of grain.

Consolidated media.

It is known, that in the consolidated rocks for minerals with covalent connection between particles (I group) is observed the growth of velocities of P waves by growing of density, but for minerals with metal connection between grains (II group) there is a fall of velocity [2] (Fig. 3). In the contrary of high porosity media, the density of consolidated ones is equal to the one of a grain, and it is loading by constrained conditions. It means that by normal deformation (normal with respect to platform of contact) of two neighbor particles lead deformations of all neighbors. Let's present a medium as a set of mesostructures, consisting of N centers of contacts and random

particle (polyhedron), which is surrounding of these neighbors. N – an average of contacts. It is possible to write the normal displacement from position of equilibrium for the central particle, which is caused by relative displacement of neighbor in the form (like (2)).
By deforming processes where there is a possibility to move of matter into of pore space, the displacement of one side is not causes a displacement of other sides (surplus of mass is going to the pores). It is obvious, that in the constrained conditions there is an influence of deformation of all sides. Clearly, that the exact decision of such problem is represented difficult enough. However under condition of small displacement of sides in comparison with the size of a grain, it is possible to take into account approximate expression:

$$\delta_{n\,i} = (N-2)\delta_{n\,i0}/(N-1) + \Sigma\delta_{n\,i0}/(N-1), \qquad (11)$$

where symbol Σ means the sum with respect to all contacts, $\delta_{n\,i0}$ is displacement, with caused by *two* neighbor particles, while δ_{ni} is displacement due to deformation of *all* sizes.

In order to write equations of motion it is necessary to know two simple microphysical constants only (A and B), which are determine of normal and tangent forces:

$$F_n = A\delta_n \qquad (12)$$

$$F_\tau = B\delta_\tau \qquad (13)$$

where δ_n δ_τ and means normal and tangent displacements.

It is evident that A>0. As to sign of B there is not simple situation. It is due to two methods, which provide of tangent displacement:

1) The shift (the sign of B is positive).
2) The rotation of neighbor with respect to the center of a contact platform (B < 0)

Physical reasons lead that B > 0 for nonmetals and B < 0 for metals. Besides of it $|B/A|$ should grow by growing of a grain material density.
For creation of equations of motion it is necessary to use a procedure of the averaging, like described above, which are represent 3-D equations of elasticity. Velocities of P and S waves take a form:

$$V_p^2=(3A1+B)\,N/(180\pi\rho d) \qquad (14)$$

$$\gamma^2 = (V_s/V_p)^2= \mu/(\lambda+2\mu)= (A1+3B)/(3A1+B) \qquad (15)$$

where A1= A(N–2)/(N–1), d – average diameter of grain, ρ - density of medium.

It is known, that in materials with covalent type of connections between grains there is an increase of P wave velocity by growing of density [2] (Fig 3). Formula (14) explains this effect because $|B/A|$ increases by growth of density for all materials, and the sign of B is positive for covalent materials. As to metals, the sign of B is negative value. It means that the P wave velocity decreases by growing of density according to experimental observations. The limited value of B=A/2 corresponds γ (V_s/V_p) ≈ 0.8 (Poisson ratio is negative). This procedure of construction of the equations does not deny an opportunity of existence of media with abnormal high ratio of S and P waves velocities for nonmetals with high density. In [3] is shown the experimental data about velocities of P and S waves in glass from BeF_2. There is $\gamma \approx 0.8$. It is possible that it is not a mistake of measurement, and Poisson ratio in this material it is really negative.

CONCLUSION

It is established that the ratio V_s/V_p depends on microstructure of particles. For sandy particles, which are acting by Hertz law this ratio is $1/\sqrt{3}$, while for clay particles of cellular structure, which are acting by dipole interactions the mentioned ratio is $1/\sqrt{11}$. For consolidated media this ratio depends on two microphysical constants only.

References

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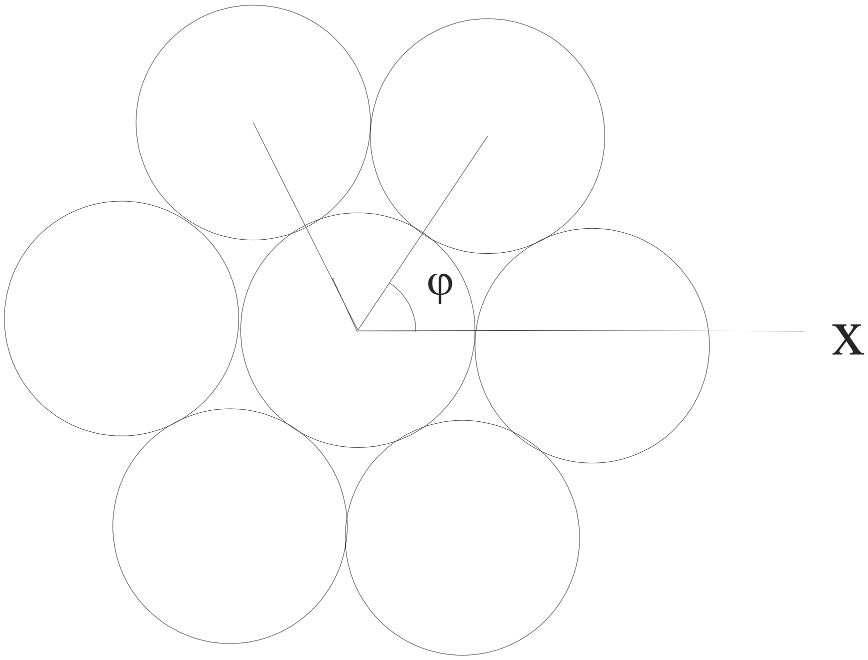

Fig.1. Microstructure of plane section of sandy medium with identical particles.

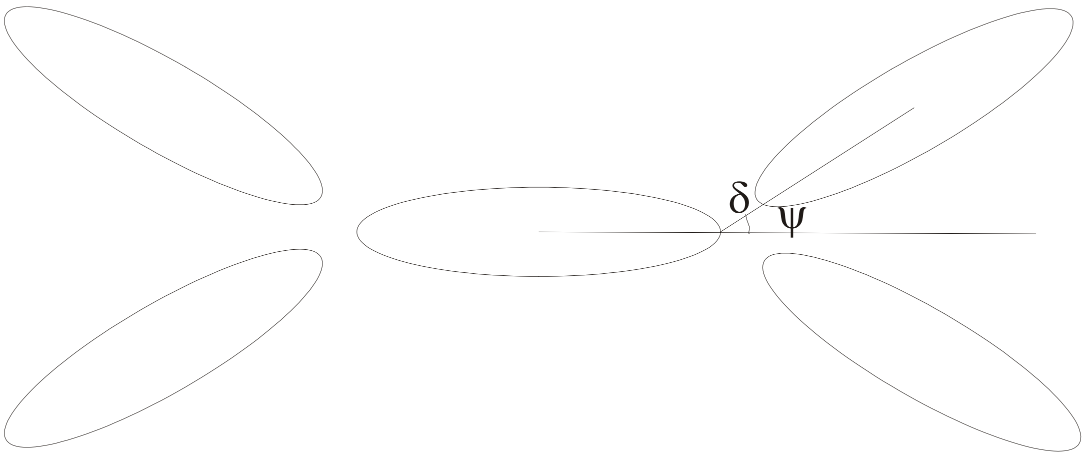

Fig.2. Microstructure of clay particles, which are acting by dipole interactions. δ is distance between particles. ψ is a solid angle between plane sections of particles.

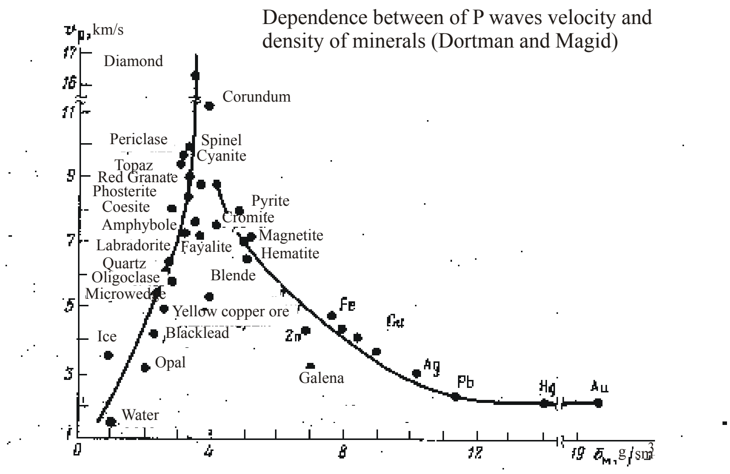

Fig.3. There is shown two kinds of materials behavior by growing of the density. The first kind – left curve, the second kind – right curve.